

Dilepton Production in Nucleon-Nucleon Interactions *

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Abstract

Starting from a realistic one-boson-exchange-model fitted to the amplitudes of elastic nucleon-nucleon scattering and the process $NN \rightarrow N\Delta$ we perform a fully relativistic and gauge invariant calculation for the dilepton production in nucleon-nucleon collisions, including the important effect of propagating the Δ -resonance. We compare the results of our calculations with the latest experimental data on dilepton production. We also show how to implement various electromagnetic formfactors for the hadrons in our calculations without loosing gauge-invariance and discuss their influence on dilepton spectra.

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1 INTRODUCTION

Heavy-ion reactions of intermediate energies up to a few GeV/u offer the unique possibility to produce chunks of nuclear matter of densities of $2 - 3 \rho_0$ [1]. It is predicted by various models [2] that at these densities first effects from the restoration of chiral symmetry, one of the fundamental symmetries of QCD, should appear. In particular, a lowering of the masses of the vector mesons has been predicted [3] that could possibly be observed in measurements of dilepton spectra. In ref.[4] we have investigated the observable consequences of such changes.

In this paper we study the elementary production process for dileptons in nucleon-nucleon collisions, since the quantitative understanding of this process is the natural prerequisite to an unequivocal determination of the in-medium effects mentioned above. We also point out that the study of this process may give fundamental information on the electromagnetic formfactor of the nucleon in the timelike region around the vector meson masses that is otherwise not accessible [1].

For the production of photons the relevant cross sections were presented in [5]. Similar calculations were performed for dileptons, but the effect of resonances was neglected [6, 7, 8]. In ref.[8] calculations quite similar to our calculations for the nucleon contribution to the cross section were performed but there only back-to-back pairs of the dileptons were considered, because in this case the phasespace for the nucleons can be integrated out in an analytic way. The cross section obtained in this way was assumed to be also valid for dileptons that are not back-to-back pairs.

In chapter 2 we will shortly present our general calculational procedure based on an effective OBE-interaction where all the parameters are determined by fitting the elastic nucleon-nucleon scattering data and the $NN \rightarrow N\Delta$ reaction. Using this set of parameters in chapter 3 our results for dilepton production will be presented.

Since all hadrons have an electromagnetic structure, we have to include formfactors in our calculation, too. How it is possible to do this without loosing gauge invariance is shown in subsection 1 of chapter 3. In the last subsection of chapter 3

we present the results of our calculations using the parameters found in chapter two. The conclusion in chapter four will end our paper.

2 Effective OBE scattering amplitude

To obtain the desired production cross sections we use covariant perturbation theory in first order in the nucleon-nucleon interaction. Particles are simply produced from the external nucleon lines or, if possible, from the internal meson lines (Fig. 1). Particle emission between subsequent NN interaction vertices are omitted in our calculation scheme, but we include the more important effects of excited Δ -resonances.

To determine all the parameters involved in our calculation we need as an essential input the nucleon-nucleon interaction. In contrast to some iterative scheme we directly fit the T -matrix by an effective nucleon-nucleon interaction based on an One-Boson-Exchange Model. For lab energies up to 400 MeV such fits were performed, using up to 16 mesons [9, 10]. Here we restrict ourselves to the most essential four mesons for fitting elastic nucleon-nucleon scattering data for lab energies in the range from 800 MeV up to 3 GeV. For these four mesons we use the following Hamiltonian:

$$\begin{aligned}
H_{int} &= g_\sigma \bar{\psi} \psi \sigma + g_\omega \bar{\psi} \gamma_\mu \psi \omega^\mu \\
&+ g_\rho \bar{\psi} \left(\gamma_\mu + \frac{\kappa \partial_\mu}{2m_N} \right) \vec{\tau} \psi \vec{\rho}^\mu \\
&+ i g_\pi \bar{\psi} \gamma_5 \vec{\tau} \psi \vec{\pi}
\end{aligned} \tag{1}$$

where $\sigma, \omega, \vec{\rho}$ and $\vec{\pi}$ denote the scalar, vector, isovector-vector and isovector-pseudoscalar fields, respectively, of the four mesons.

From the pion-nucleon interaction the NN potential

$$\begin{aligned}
V_\pi(q) &= - \left(\frac{g_\pi}{2m_N} \right)^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \\
&\times \left(\frac{\vec{\sigma}_1 \cdot \vec{q} \quad \vec{\sigma}_2 \cdot \vec{q}}{m_\pi^2 + \vec{q}^2} \right)
\end{aligned} \tag{2}$$

can be deduced. One feature of this potential is that for $\vec{q} = 0$, i.e. for the forward direction in the direct diagrams and the backward direction in the exchange diagrams,

the scattering amplitude vanishes; this leads to an unreasonable shape of the elastic scattering cross section. To avoid this, we first decompose the potential eq.(2) into the sum of a tensor part and a central part. After a Fourier-transformation of the central part we get the well-known Yukawa-interaction and an additional contact term [11].

$$V_{\pi}^C(r) = \frac{m_{\pi}^3}{12\pi} \left(\frac{g_{\pi}}{2m_N} \right)^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \quad \vec{\tau}_1 \cdot \vec{\tau}_2 \\ \times \left(\frac{e^{-m_{\pi}r}}{m_{\pi}r} - \frac{4\pi}{m_{\pi}^3} \delta(r) \right) \quad (3)$$

It is the contact term that causes the unphysical behaviour in the angular distribution. In nature this term is effectively turned off by the repulsive hard core of the nucleon–nucleon interaction, but in our scheme it has to be explicitly subtracted. This is achieved by adding the following Hamiltonian:

$$\Delta H_{int} = g_a \left(\bar{\psi} \gamma_5 \gamma_{\mu} \vec{\tau} \psi \right) \left(\bar{\psi} \gamma_5 \gamma^{\mu} \vec{\tau} \psi \right) \quad (4) \\ g_a = -\frac{1}{3} \left(\frac{g_{\pi}}{2m_N} \right)^2$$

since

$$\gamma_0 \gamma_5 \gamma_{\mu} = (-\gamma_5, \vec{\sigma}) . \quad (5)$$

Eq.(4) describes the interaction of the nucleons via a very heavy axialvector–isovector meson. For high bombarding energies and inelastic nucleon-nucleon processes this interaction generates additional terms besides the desired δ -force in coordinate space, but these are of minor importance in comparison to the influence of the δ -force. Fig. 2 shows clearly the improvement obtained when using the contact term eq.(4) in addition to a pion-potential.

To take into account the finite size of the nucleons we introduce formfactors

$$F_i(q^2) = \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q^2}, \quad (6)$$

at each strong-interaction vertex where q is the four-momentum and m the mass of the exchanged meson.

At total we have eight parameters which have to be determined by elastic nucleon-nucleon scattering data. In an effective interaction scheme it is not possible to reproduce all the data in the desired energy range from 800 MeV up to 3 GeV with energy independent parameters. Therefore we have kept the four cutoffs Λ energy independent and use for each meson a coupling constant which depends on the total c.m. energy

$$g(\sqrt{s}) = g_0 e^{-l\sqrt{s}} \quad . \quad (7)$$

With these 12 parameters $(\Lambda, g(0), l)_i$ given in Table 1 we fit the T -matrix to the relevant pp and pn data at three different laboratory energies 1.73, 2.24 and 3.18 GeV for proton-proton and proton-neutron scattering. In Fig. 3 we show the results of our calculations using these parameters in comparison with the experimental data [12]. Note that the 800 MeV-data (Fig. 3 top) were not included in our fit procedure. From this comparison we conclude that the elastic amplitudes are quite well described. We then assume that the NN-meson vertices determined by the procedure just described are correct also for half-off-shell processes.

As already mentioned, for particle production in the energy region we are interested in, it is necessary to include higher resonances. In this paper we limit ourselves to the implementation of the Δ . The Feynman diagrams for the particle production involving the Δ always look like the process $N + N \rightarrow N + \Delta$ and the decay of this Δ into a nucleon and the desired particle (pion or dilepton pair). Due to isospin conservation only isospin-1-mesons can be exchanged; in our calculation these are the pion and the rho-meson. The corresponding vertex functions are:

$$- \frac{f_{N\Delta\pi}}{m_\pi} q^\mu \vec{T} \quad \text{for the pion} \quad (8)$$

$$-i \frac{f_{N\Delta\rho}}{m_\rho} \left(q^\beta \gamma_\beta g_{\mu\alpha} - q_\mu \gamma_\alpha \right) \gamma_5 \vec{T} \quad \text{for the rho} \quad (9)$$

where q_μ denotes the momentum of the outgoing meson and \vec{T} the isospin-operator. From the decay $\Delta \rightarrow N + \pi$ we have determined the $N\Delta\pi$ -coupling constant to $f_{N\Delta\pi} = 2.13$.

The Δ propagator [13] is :

$$G_{\Delta}^{\mu\nu}(p) = -\frac{(p_{\eta}\gamma^{\eta} + m_{\Delta})}{p^2 - m_{\Delta}^2} \left(g^{\mu\nu} - \frac{1}{3}\gamma^{\mu}\gamma^{\nu} - \frac{2}{3m_{\Delta}^2}p^{\mu}p^{\nu} - \frac{1}{3m_{\Delta}}(\gamma^{\mu}p^{\nu} - p^{\mu}\gamma^{\nu}) \right) \quad (10)$$

Concerning this propagator there is a some confusion in the literature, ref.[14] used p^2 in the projector part of the propagator instead of m_{Δ}^2 . This, however, leads to an unreasonable behaviour, since the projector develops a pole for off-shell Δ 's with $q^2 = 0$, which shows up as a pole in the differential cross section for pion production.

The mass of the Δ in the denominator is modified by an imaginary width $m_{\Delta} \rightarrow m_{\Delta} - i\Gamma/2$ due to the fact that Δ 's are not stable particles. Nonrelativistic calculations of the Δ selfenergy lead to a k^3 dependence of the width [15] where k is the pion momentum in the rest frame of the Δ . We take [16]

$$\begin{aligned} \Gamma(\mu^2) &= \Gamma_0 \left[\frac{k(\mu^2)}{k(m_{\Delta}^2)} \right]^3 \frac{k^2(m_{\Delta}^2) + \kappa^2}{k^2(\mu^2) + \kappa^2} \\ k(\mu^2) &= \sqrt{\frac{(\mu^2 + m_N^2 - m_{\pi}^2)^2}{4\mu^2} - m_N^2} \end{aligned} \quad (11)$$

where μ is the mass of an off-shell Δ . The constant Γ_0 is the free width of 0.12 GeV and κ is fixed to 0.16 GeV.

As in the case of nucleon-nucleon scattering (6) we use formfactors for the nucleon- Δ vertex but now of dipole form [17] in order to keep the self-energy of the nucleon finite (a monopole formfactor would not change any of the results reported below if we use a somewhat smaller cut-off).

$$\begin{aligned} F_i^*(q^2) &= \left(\frac{\Lambda_i^{*2} - m_i^2}{\Lambda_i^{*2} - q^2} \right)^2 \\ i &= \pi, \rho \quad . \end{aligned} \quad (12)$$

The three new parameters f_{ρ} , Λ_{π}^* and Λ_{ρ}^* are fitted to the existing data [16] for mass-differential cross sections for $N + N \rightarrow N + \Delta$ in the energy range 1 - 2.5 GeV. For the calculation of the Δ -production we take also into account the decay width of the

Δ by multiplying

$$\rho(\mu) = \frac{1}{\pi} \frac{m_{\Delta} \Gamma(\mu^2)}{(\mu^2 - m_{\Delta}^2)^2 + m_{\Delta}^2 \Gamma^2(\mu^2)} \quad (13)$$

to the cross section obtained from the Feynman diagram.

In a fit to experimental data of $pp \rightarrow n\Delta^{++}$ at 3 energies (970 MeV, 1.48 GeV, 2.02 GeV) we have obtained

$$\begin{aligned} \Lambda_{\pi}^* &= 1.421 \text{ GeV} \\ \Lambda_{\rho}^* &= 2.273 \text{ GeV} \\ f_{N\Delta\rho} &= 7.4 \quad . \end{aligned} \quad (14)$$

We have found that in this case a good fit could be obtained already without a \sqrt{s} -dependence, except at the highest energy. The results of our fits in comparison to the experimental data [16] at 0.970, 1.48 and 2.02 GeV are shown in Fig. 4. Up to 2.0 GeV we have found a destructive interference between π - and ρ -meson, above 2.0 GeV there is a constructive interference. We then conclude that now also the $N\Delta$ -meson vertices are determined; as for the case of the NN -meson vertices we assume them to be correct also for half-off-shell-processes.

In order to check this hypothesis we perform calculations for the pion production in nucleon-nucleon collisions as a first test to verify our potentials. We compare our calculations with experimental data for five-times-differential cross sections for $pp \rightarrow pn\pi^+$ [18]. Figure 5 shows good agreement between experimental data and our calculation.

For smaller energies the fitted coupling constant are not very energy dependent. We thus used the coupling for 800 MeV also for smaller energies (400 MeV – 800 MeV). With these parameters we also reproduce the total cross sections for elastic nucleon–nucleon scattering within 20%.

We take this as a justification to calculate with these parameters also the total cross sections for the $pp \rightarrow pn\pi^+$ process for lower energies. Figure 6 shows that we get a good agreement with experiment (experimental data are taken from [12]). Using

only Feynman graphs with propagating Δ we get much too low cross sections for small energies; there the production over the nucleon pole is obviously very important.

3 DILEPTONS

After having convinced ourselves in the last section that the effective OBE model gives a good description of elastic scattering and pion production amplitudes, we now use the interaction vertices determined there for a calculation of dilepton production in elementary nucleon–nucleon collisions. In this calculation we first neglect any electromagnetic formfactors of the nucleons and determine the coupling constant $f_{N\Delta\gamma}$ from the γ –decay branch of the Δ . The vertex function for this process is:

$$-i \frac{f_{N\Delta\gamma}}{m_\pi} \left(q^\beta \gamma_\beta g_{\mu\alpha} - q_\mu \gamma_\alpha \right) \gamma_5$$

$$f_{N\Delta\gamma} = 0.32 \quad , \tag{15}$$

where q is the momentum of the photon in this case. The coupling constant has been obtained from the experimental partial width of the Δ of 0.6% for the decay into a real photon.

We also include the coupling to the anomalous magnetic moment of the nucleon. A consequence of this is that now also the neutron lines can contribute to the radiation of dileptons. Both the Δ resonance and the radiation from neutron lines were not included in our earlier calculations [7] nor in those of Haglin et al. [6]. That the latter becomes more important for higher photon-energies was already found in our calculations of real bremsstrahlung [5].

Figs. 7 and 8 show the invariant mass spectra for dilepton production both for pp and pn collisions at the two bombarding energies 1.0 and 2.1 GeV. In all cases the dominant contribution arises from the resonance amplitude where the intermediate, off-shell particle is a Δ ; in fact the total yield is almost equal to the contribution of the Δ amplitude alone. The pn amplitude is for these graphs about a factor 2 – 3 larger than for the pp reactions; this factor is mainly determined by different isospin

factors.

The graphs involving only intermediate nucleon lines are for the pn reactions significantly larger than for pp (factors 6 and 4 at 1 and 2.1 GeV, respectively). This is due to the destructive interference between the radiation from both particle in the case of pp scattering. The total yield from pn is larger than that from pp by only about a factor of 4 at 1 GeV and a factor of 2.5 at 2.1 GeV (both at $M = 0.2$ GeV).

At first sight the mass-spectra of the nucleon graphs and the Δ graphs alone look very similar. However, a closer inspection shows that that at 1 GeV both merge at the high end of the mass spectrum. In this situation a sizeable interference of both contributions also takes place. Note also that the increase of the bombarding energy leads to a hardening of the spectra, but not to an increase of the value at small M .

In a first comparison with experimental data we now look at the latest results of the BEVALAC group[19, 20] for the ratio of the pd- to pp-dilepton yield. In Fig. 9 we show this ratio for lab energies from 1.03 GeV up to 4.9 GeV, the experimental data[19, 20] and the results of our calculations (solid line). The calculation for the deuteron was performed by summing incoherently over the total pn and pp cross sections. Since our model gives the correct scattering amplitudes only up to 3.0 GeV we omit the calculation for 4.9 GeV and show the data for completeness only. It is seen that the calculation reproduces the general trend of the pd/pp ratio very well. In particular it is able to describe the absolute value of the ratio as well as its mass-dependence correctly; also the rise with M towards the kinematical limit at the lower bombarding energies is obtained, although maybe somewhat too small (at 1.26 GeV).

A closer inspection of the reasons for this behavior shows that we get a destructive interference between the nucleon- and the Δ -contributions for the pp reaction which in the high mass region is as large as the nucleon contribution (see Fig. 7–8), the largest one there. For the pn reaction at the kinematical limit we obtain also a destructive interference which is as large as the in the pp case, but for all invariant masses smaller than the largest of the individual contributions by at least a factor

2–3 so that it has virtually no effect in the pn case.

Figure 10 shows the results of the calculation, together with data, for the $p + {}^9\text{Be}$ reaction [21]. Note that for this comparison we have made the assumption that all dileptons are radiated incoherently and that none of the Δ 's are reabsorbed before they can emit a dilepton pair. This means that the cross sections from pn– as well as pp–scattering are summed up, weighted by the number of neutrons or protons in the nucleus, respectively.

The efficiency of the detector is included in the integration over the phasespace–volume by using the latest filter–routine from the DLS collaboration[29]. We use the same filter for all the various dilepton–sources. Below 300 MeV invariant mass of the dilepton pair the filter can decrease the yield by factors of two or three; in this energy range we fail to describe the data. In particular, we cannot describe the dip in the data around 150 MeV.

While we overestimate the data at 1 GeV we obtain good agreement at 2.1 GeV, except for invariant masses around the ρ –mass (see Fig. 10), where the calculated values are somewhat too low. Note also that the 2.1 GeV data contain a contribution from η –Dalitz decay; we take this contribution from ref.[22].

While the resonance terms give the largest contributions for the elementary process, their influence may be overestimated when scaling this cross section up to the experimental case of $p + {}^9\text{Be}$, because of a possible reabsorption of the Δ by another nucleon before emitting the virtual photon, the Pauli–blocking of the final state and possible shadowing effects. A comparison of our calculations with experiments on the elementary process $pp \rightarrow ppe^+e^-$ would be free of such uncertainties.

3.1 Gauge invariance

Besides the contribution from bremsstrahlung, our calculation also includes dilepton production from the charged internal meson lines (Fig. 1c). For this internal radiation, from the pions, for example, we could use the experimentally well established

electromagnetic formfactor of the pion [11] which then would lead to an enhancement of the cross section just in the region of the ρ -mass. This procedure, however, is not gauge-invariant, since current conservation requires

$$q^\mu J_\mu = 0 \quad , \quad (16)$$

so that at a first sight the nucleons must have the same electromagnetic structure as the pions. Thus we are lead to the strong VMD hypothesis, i.e. all hadrons couple to photons by an intermediate ρ -meson. The dipole shape of the spacelike electromagnetic formfactor of the nucleon shows that this is too strong a restriction. This section shows how to keep gauge invariance with different formfactors for the various hadrons.

The problem of losing gauge invariance when using electromagnetic formfactors arises because we have so far not used the complete vertex function for the half-off-shell photon production vertices. The full photon vertex for the nucleon is [23]:

$$\begin{aligned} \Gamma_\mu(p', p) = & \\ & e \left(\Lambda^+(p') \left[F_1^{++} \gamma_\mu + F_2^{++} \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} + F_3^{++} q_\mu \right] \Lambda^+(p) \right. \\ & + \Lambda^+(p') \left[F_1^{+-} \gamma_\mu + F_2^{+-} \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} + F_3^{+-} q_\mu \right] \Lambda^-(p) \\ & + \Lambda^-(p') \left[F_1^{-+} \gamma_\mu + F_2^{-+} \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} + F_3^{-+} q_\mu \right] \Lambda^+(p) \\ & \left. + \Lambda^-(p') \left[F_1^{--} \gamma_\mu + F_2^{--} \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} + F_3^{--} q_\mu \right] \Lambda^-(p) \right) \end{aligned} \quad (17)$$

where p and p' are the initial and final nucleon four-momenta of the nucleon, which can be off-shell. The formfactors F depend on the three variables p^2, p'^2 and the photon momentum squared, q^2 . The indices $+$ and $-$ denote the positive and negative energy states of the nucleon, the quantities

$$\begin{aligned} \Lambda^\pm(p) &= \frac{\pm p_\mu \gamma^\mu + W}{2W} \\ W &= (p^2)^{1/2} \end{aligned} \quad (18)$$

are the corresponding projection operators. By using the Ward-Takahashi identity

(WTI) [24]

$$\begin{aligned} q_\mu \Gamma^\mu(p', p) &= ee_N [S^{-1}(p') - S^{-1}(p)] \\ e_N &= (1 + \tau_3)/2, \end{aligned} \quad (19)$$

where $S(p)$ is the propagator of the particle, we find a relation between F_1 and F_3 :

$$\begin{aligned} \Lambda^\pm(p') - F_3^{\pm\pm} \Lambda^\pm(p) &= \\ \frac{1}{q^2} \Lambda^\pm(p') \left(-q_\mu \gamma^\mu F_1^{\pm\pm} + e_N [S^{-1}(p') - S^{-1}(p)] \right) \Lambda^\pm(p), \end{aligned} \quad (20)$$

where e_N distinguishes between protons and neutrons. For given formfactors F_1 we can thus satisfy the WTI by choosing the F_3 formfactors according to eq.(20).

That this will restore gauge-invariance can be easily seen if we check eq.(16) for pn-bremstrahlung by an exchange of an *uncharged* meson. The various contributions to the four-divergence are (see Fig. 1):

Diagram 1a :

$$\begin{aligned} q_\mu J^\mu &= \bar{u}(p_3) [\Gamma_{NN} S(p_1 + q) q_\mu \Gamma^\mu] u(p_1) \\ &\times \bar{u}(p_4) \Gamma_{NN} u(p_2) D(p_4 - p_2) \\ &= \bar{u}(p_3) [\Gamma_{NN} S(p_1 + q) S^{-1}(p_1 + q)] u(p_1) \\ &\times \bar{u}(p_4) \Gamma_{NN} u(p_2) D(p_4 - p_2) \\ &= \bar{u}(p_3) \Gamma_{NN} u(p_1) \bar{u}(p_4) \Gamma_{NN} u(p_2) D(p_4 - p_2) \end{aligned}$$

Diagram 1b :

$$\begin{aligned} q_\mu J^\mu &= \bar{u}(p_3) [q_\mu \Gamma^\mu S(p_3 - q) \Gamma_{NN}] u(p_1) \\ &\times \bar{u}(p_4) \Gamma_{NN} u(p_2) D(p_4 - p_2) \\ &= \bar{u}(p_3) [-S^{-1}(p_3 - q) S(p_3 - q) \Gamma_{NN}] u(p_1) \\ &\times \bar{u}(p_4) \Gamma_{NN} u(p_2) D(p_4 - p_2) \\ &= -\bar{u}(p_3) \Gamma_{NN} u(p_1) \bar{u}(p_4) \Gamma_{NN} u(p_2) D(p_4 - p_2) \end{aligned}$$

$$\text{Diagram 1a} + \text{Diagram 1b} = 0 \quad \text{q.e.d.} \quad (21)$$

Here $D(p)$ is the meson propagator and $S(p)$ that of a nucleon; eq. (19) has been exploited in these expressions.

In the case of *charged* meson exchange one has to interchange p_3 and p_4 and use in diagram 1b $D(p_1 - p_4)$ instead of $D(p_4 - p_2)$. In this case the sum of both diagrams does not vanish. This is because the meson can radiate a photon, too. Thus we get the additional diagram 1c:

$$\begin{aligned}
q_\mu J^\mu &= \bar{u}(p_4) \Gamma_{NN} u(p_1) \bar{u}(p_3) \Gamma_{NN} u(p_2) \\
&\times D(p_1 - p_4) \Gamma_{(\pi)\mu} q^\mu D(p_3 - p_2) \\
&= \bar{u}(p_4) \Gamma_{NN} u(p_1) \bar{u}(p_3) \Gamma_{NN} u(p_2) \\
&\times D(p_1 - p_4) \left[D^{-1}(p_3 - p_2) - D^{-1}(p_1 - p_4) \right] D(p_3 - p_2) \\
&= \bar{u}(p_4) \Gamma_{NN} u(p_1) \bar{u}(p_3) \Gamma_{NN} u(p_2) \\
&\times (D(p_1 - p_4) - D(p_3 - p_2)) \quad .
\end{aligned} \tag{22}$$

The sum of all three diagrams vanishes.

In our actual calculation the hadronic vertices contain also strong formfactors depending on the four-momentum of the exchanged meson (eq.6). For uncharged mesons this means to multiply all the above diagrams simply by the same number $F_i(p_4 - p_2)$ (6), thus keeping gauge invariance. In the case of charged meson exchange, however, the four-momentum of the meson changes in diagram 1c. This change can be taken into account by an additional factor in eq.(22) [6, 25]:

$$1 + \frac{m_{meson}^2 - q_1^2}{\Lambda^2 - q_2^2} + \frac{m_{meson}^2 - q_2^2}{\Lambda^2 - q_1^2} \tag{23}$$

which can be interpreted as the radiation of photons by the heavy cutoff-particles which carry the same quantum numbers as the corresponding meson.

In [23, 26] the same result is obtained in a different way. There the strong interaction formfactors and the free meson propagator are used together as an effective meson propagator. Then the WTI for the meson is fulfilled for this new propagator. This leads to electromagnetic formfactors for the meson-gamma vertex, which agree

with eq.(23). Thus from this way of gauging the strong formfactors it is evident that there is the possibility to use a given electromagnetic formfactor for the meson and a different one for the nucleon and still to fulfil the WTI.

3.2 Formfactors

Coming back to the formfactors, it was shown in eq.(17) that all formfactors depend on three kinematical variables. The dependence of the on-shell formfactors on the invariant mass M of the dileptons can be obtained for the hadrons from various experiments, at least for some specific region of M .

For the pion we use the electromagnetic formfactor, experimentally well determined from $\pi^+\pi^-$ -annihilation [11]

$$\begin{aligned} F_\pi(M^2) &= \frac{m_V^2}{m_V^2 - M^2} \\ m_V &= 770 \text{ MeV} \quad . \end{aligned} \tag{24}$$

This enters into the calculation of the contribution of diagram 1c.

Unfortunately, for the nucleons information on the formfactor is only available for $M^2 < 0$ (from electron-proton scattering) and for $M > 2m_N$ (from $e^+e^- \leftrightarrow p\bar{p}$). The region $0 \leq M \leq 2m_N$ is for the nucleons not accessible in any on-shell experiment; this is unfortunate since this is just the interesting region that includes the vector-meson masses and is thus crucial for establishing the validity of vector meson dominance for the nucleon.

In order to test the sensitivity of our results to the formfactors of the nucleon we use two different models. In the first we extrapolate the model of ref.[27], which gives an excellent fit in the space-like domain, into the timelike region. In that model the formfactor arises from the meson cloud surrounding the quark core as well as from the core itself.

$$\begin{aligned} F_C(M^2) &= \frac{\Lambda^2}{\Lambda^2 - M^2} \\ F_R(M^2) &= \frac{2m_V^4}{(m_V^2 - M^2)(2m_V^2 - M^2)} \end{aligned}$$

$$\begin{aligned}
F_1^{proton}(M^2) &= \frac{1}{2}(F_C(M^2) + F_R(M^2)) \\
F_2^{nucleon}(M^2) &= F_R(M^2) \\
\Lambda &= 893 \text{ MeV} \\
m_V &= 650 \text{ MeV} \quad .
\end{aligned} \tag{25}$$

In the actual calculation the F_2 -formfactor is multiplied with the proper anomalous magnetic moment for a proton or a neutron. For the Δ we take for simplicity the same formfactor F_2 as for the nucleon.

In the second model we use the same formfactor as for the pion (24) also for the nucleon and the Δ . This would correspond to strict vector meson dominance.

By adding to the masses and cutoffs a typical width of 100–150 MeV we avoid the pole structure of these formfactors and make the latter complex. Note, however, that this procedure does not include any imaginary parts resulting from other possible decay channels. Our results depend especially in the pole region on the value we take for the width, but if we believe in VMD this width should not be too different from the width of the rho-meson. The formfactors for the pion and nucleon are shown in Fig. 11. Note that the model of ref.[27] exhibits a strange structure in F_1 and F_2 in the high mass region, which may simply indicate that this model has been extrapolated beyond its limits of validity.

In order to get a feeling for the influence of the formfactors on the spectra we now make two assumptions, namely that first these formfactors can also be applied to the half-off-shell vertices appearing in dilepton production without any off-shell corrections and, second, that all the formfactors F^{++} , F^{+-} and F^{-+} are the same (the F^{--} -formfactors do not appear in the process treated here).

The results of this calculation are shown in Fig. 12 for the bombarding energy of 2.1 GeV. Since we are interested here only in the effects of the electromagnetic formfactors we do not show the η -Dalitz decay contribution (cf. Fig. 10) again.

In the region of interest, $M = 600$ MeV or higher, there is no contribution from the η -decay. Thus it might be possible to extract an estimate for the hadronic

formfactors from the existing data . In Fig. 12 the dotted line corresponds to our calculation without any formfactor; it underestimates the data in the mass region around 600 MeV by about a factor of 3. Fig. 8 shows that the contribution of the propagating Δ provides the dominant amplitude for this process. By using the formfactors (from ref.[27]) we now would overestimate the data drastically in the meson mass region between 600 and 700 MeV (dashed line in Fig. 12). If we use the strict VMD formfactors we obtain the dashed curve which shows a shoulder-like structure which is not seen in the data.

These results show that the cross section for the production of dileptons of high invariant mass in nucleon–nucleon collisions is very sensitive to the electromagnetic formfactors of the nucleon and the Δ . The fact that the calculations with the formfactors included overestimate the measured cross section in the vector meson mass region and that the data - within their present accuracy - do not show any indication of an enhancement in the vector meson mass region might have several reasons. First, neglecting the half-off-shell corrections may not be correct, second, our setting $F^{++} = F^{-+} = F^{+-}$ may not be justified and, third, unitarity constraints may play a role so that the formfactors really are complex in this region.

4 CONCLUSIONS

In this paper have presented calculations of the cross sections for dilepton production in nucleon-nucleon collisions in a relativistic scheme. As a major input for all these calculations a covariant amplitude based on an one-boson-exchange-model was presented, which allows to describe experimental data for $NN \rightarrow NN$ and $NN \rightarrow N\Delta$ for lab energies from 800 MeV up to 2.5 GeV and also works well for pion production. We have then used the vertices so determined in a calculation of the elementary dilepton production cross section and have discussed how to include different electromagnetic formfactors for the hadrons while maintaining gauge invariance whatever theoretical model for these formfactors we insert.

In this framework we can describe the latest data on dilepton production in pd and pp reactions, both in their absolute magnitude and in their trends. Earlier data for $p + Be$ can also be described reasonably well, although here discrepancies remain, mainly at the lowest energy.

We have then shown the importance of the electromagnetic formfactors of the hadrons for the dilepton production in nucleon-nucleon and nucleon-nucleus collisions. These reactions are a unique tool to study the electromagnetic formfactor of the nucleons in the timelike region around the vector-meson masses and are thus essential to check the validity of vector-meson dominance directly, at least for the half-off-shell formfactors. In view of the adhoc nature of the formfactors used here this can presently be only an explorative study which, however, points to the need to include the half-off-shell effects of the electromagnetic formfactors and their imaginary parts in the calculations.

Precise data on the elementary process would clearly help to put constraints on these formfactors. In particular experiments are necessary where also the hadrons in the final state are detected. Only in these experiments it might be possible to disentangle all the various contributions coming from pre- or postemission diagrams, from nucleon- or Δ contribution and from direct- or exchange-diagrams.

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Table

	$\frac{g^2}{4\pi}$	l	m [GeV]	Λ [GeV]
π	12.562	0.1133	0.138	1.005
σ	2.340	0.1070	0.550	1.952
ω	46.035	0.0985	0.783	0.984
ρ	0.317	0.1800	0.770	1.607
	$\kappa=6.033$			

Table 1: Coupling constants used in the calculations.

Figure captions

Fig. 1 Feynman diagrams for emission of the photon after (a), before (b) and during (c) the nucleon-nucleon interaction. The double line denotes either an off-shell nucleon or a Δ resonance.

Fig. 2 Differential cross section for proton-neutron scattering by using only pion-exchange potential (dashed line) and by using the contact interaction (4) in addition to the pion (solid line).

Fig. 3 Result of our calculations for nucleon-nucleon scattering at 800 MeV, 1.7 GeV and 2.24 GeV lab energy in comparison to experimental data[12]. Solid line: proton-proton, dashed line: proton-neutron. The solid dots give the pp-data while the stars denote the pn data [12].

Fig. 4 Result of our calculations for the mass-differential cross section for the reaction $pp \rightarrow n\Delta^{++}$ at 0.970, 1.5 and 2.02 GeV lab energy in comparison to experimental data[16]. μ is the mass of the Δ .

Fig. 5 Result of our calculations for the five-times differential cross section for pion production in proton-proton reactions at 800 GeV lab energy in comparison to experimental data[18]. Solid line: all diagrams, dashed line: only resonance diagrams.

Fig. 6 Result of our calculations for the cross section of pion production in proton-proton scattering in comparison to experimental data[12].

Fig. 7 Result of our calculations for the cross section of dilepton production in proton-nucleon scattering at 1.0 GeV. The diamonds gives the contribution of non-resonance diagrams; the resonance contribution to this reaction is given by the crosses(+). The squares and the crosses(x) gives the positive, respectively negative interference between resonances and non-resonance amplitudes. The coherent sum of all contributions is given by the solid line. The individual contributions for the proton-proton reaction are shown in a), for the proton-neutron reaction in b).

Fig. 8 Result of our calculations for the cross section of dilepton production in proton-nucleon scattering at 2.1 GeV. For further details see figure 7.

Fig. 9 Results of our calculations (solid lines) for the pd/pp – ratio of the dilepton yield for energies from 1.03 GeV to 4.9 GeV in comparison with the experimental data from [20]. M is the invariant mass of the dilepton pair.

Fig. 10 Result of our calculations for the cross section of dilepton production in p-Be scattering for 1.0 and 2.1 GeV, including an experimental filter. The experimental data are taken from ref.[21]. For the reaction at 2.1 GeV we show our calculations with (solid line) and without (dashed line) the eta-contributions taken from ref.[22].

Fig. 11 Absolute value of the formfactors for pion (dash-dotted), F_1 -proton (solid) and F_2 -proton (dashed).

Fig. 12 Effects of electromagnetic formfactors on the dilepton invariant mass spectra in proton-Be scattering at 2.1 GeV in comparison to experimental data. The dotted line shows the result of our full calculation without any formfactors. For the solid line we include the electromagnetic formfactor for the proton, pion and Δ of ref.[27]. The dashed curve gives the result using one and the same formfactor (eq. 24) for all three particles. The data are from ref.[21] for the p-Be reaction.

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